Tema International School

Mathematics Scope, Sequence, and Benchmarks

PYP Learner Profile

Inquirers: We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.

Knowledgeable: We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.

Thinkers: We exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.

Communicators: We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

Principled: We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

Open-minded: We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.

Caring: We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.

Risk-takers: We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

Balanced: We understand the importance of balancing different aspects of our lives—intellectual, physical, (spiritual) and emotional— to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live

Reflective: We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

Introduction to the Mathematics Scope, Sequence and Benchmarks

Philosophy Statement

We believe everyone is a mathematician and strive towards instilling self-belief in every learner by supporting students to have a growth mindset. We develop resilience, creativity and knowledge through valuing the learning that comes from solving challenging problems and constructing meaning through active learning. We believe that linking knowledge and concepts to real-life contexts, providing opportunities to interact with manipulatives and to engage in conversations with others, is paramount to learning mathematics. Being numerate means using mathematical concepts and skills involving number, data handling, shape and space, pattern and function, and measurement to solve problems in daily life.

Purpose of this document

The Mathematics Scope, Sequence and Benchmark document is used to inform our learning community about the learning of mathematics at TIS. It provides the scope of concepts, knowledge and skills that are taught and assessed, the sequence in which these concepts, knowledge and skills are taught and assessed by Grade level, and what benchmarks, or criteria, all students are expected to achieve by the end of the academic level.

TIS teachers use this document as a basis for their planning, teaching and assessment of student learning. Students and parents can use this document as an overview of mathematical learning, and the TIS administration and school board can use this document as a basis for measuring student growth and school success.

This document is based on the International Baccalaureate Primary Years Programme Scope and Sequence and some aspects of the 2014 British National Curriculum for Mathematics.

How children learn mathematics

It is important that learners acquire mathematical understanding by constructing their own meaning through ever-increasing levels of abstraction, starting with exploring their own personal experiences, understandings and knowledge. Additionally, it is fundamental to the philosophy of the PYP that, since it is to be used in real-life situations, mathematics needs to be taught in relevant, realistic contexts,

rather than by attempting to impart a fixed body of knowledge directly to students. How children learn mathematics can be described using the following stages (see figure 1).
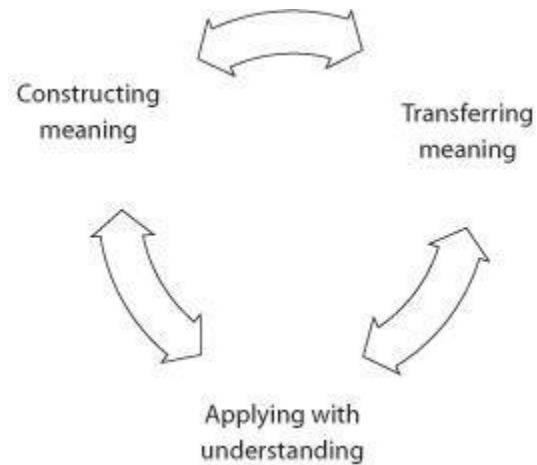
Figure 1

How children learn mathematics

Constructing meaning about mathematics

Learners construct meaning based on their previous experiences and understanding, and by reflecting upon their interactions with objects and ideas. Therefore, involving learners in an active learning process, where they are provided with possibilities to interact with manipulatives and to engage in conversations with others, is paramount to this stage of learning mathematics.

When making sense of new ideas all learners either interpret these ideas to conform to their present understanding or they generate a new understanding that accounts for what they perceive to be occurring. This construct will continue to evolve as learners experience new situations and ideas, have an opportunity to reflect on their understandings and make connections about their learning.

Transferring meaning into symbols

Only when learners have constructed their ideas about a mathematical concept should they attempt to transfer this understanding into symbols. Symbolic notation can take the form of pictures, diagrams, modelling with concrete objects and mathematical notation. Learners should be given the opportunity to describe their understanding using their own method of symbolic notation, then learning to transfer them into conventional mathematical notation.

Applying with understanding

Applying with understanding can be viewed as the learners demonstrating and acting on their understanding. Through authentic activities, learners should independently select and use appropriate symbolic notation to process and record their thinking. These authentic activities should include a range of practical hands-on problem-solving activities and realistic situations that provide the opportunity to demonstrate mathematical thinking through presented or recorded formats. In this way, learners are able to apply their understanding of mathematical concepts as well as utilize mathematical skills and knowledge.

As they work through these stages of learning, students and teachers use certain processes of mathematical reasoning.

- They use patterns and relationships to analyse the problem situations upon which they are working.

- They make and evaluate their own and each other's ideas.

- They use models, facts, properties and relationships to explain their thinking.

- They justify their answers and the processes by which they arrive at solutions.

In this way, students validate the meaning they construct from their experiences with mathematical situations. By explaining their ideas, theories and results, both orally and in writing, they invite constructive feedback and also lay out alternative models of thinking for the class. Consequently, all benefit from this interactive process.

Mathematics in a transdisciplinary programme

Wherever possible, mathematics should be taught through the relevant, realistic context of the units of inquiry. The direct teaching of mathematics in a unit of inquiry may not always be feasible but, where appropriate, prior learning or follow-up activities may be useful to

help students make connections between the different aspects of the curriculum. Students also need opportunities to identify and reflect on "big ideas" within and between the different strands of mathematics, the programme of inquiry and other subject areas.

Links to the transdisciplinary themes should be explicitly made, whether or not the mathematics is being taught within the programme of inquiry. A developing understanding of these links will contribute to the students' understanding of mathematics in the world and to their understanding of the transdisciplinary theme. The role of inquiry in mathematics is important, regardless of whether it is being taught inside or outside the programme of inquiry. However, it should also be recognized that there are occasions when it is preferable for students to be given a series of strategies for learning mathematical skills in order to progress in their mathematical understanding rather than struggling to proceed.

The structure of the mathematics scope, sequence, and benchmarks

This scope and sequence aims to provide information for the whole school community of the learning that is going on in the subject area of mathematics. It has been designed in recognition that learning mathematics is a developmental process and that the phases a learner passes through are not always linear or age related. For this reason the content is presented in continuums for each of the five **strands** of mathematics—data handling, measurement, shape and space, pattern and function, and number. For each of the strands there is a strand description and a set of **overall expectations**. The overall expectations provide a summary of the understandings and subsequent learning being developed for each phase within a strand.

The content of each continuum has been organized into four **phases** of development, with each phase building upon and complementing the previous phase. The continuums make explicit the **conceptual understandings** that need to be developed at each phase. Evidence of these understandings is described in the behaviours or **learning outcomes** associated with each phase and these learning outcomes relate specifically to mathematical concepts, knowledge and skills.

The learning outcomes have been written to reflect the stages a learner goes through when developing conceptual understanding in mathematics—**constructing meaning**, **transferring meaning into symbols** and **applying with understanding** (see figure 1).To begin with, the learning outcomes identified in the constructing meaning stage strongly emphasize the need for students to develop understanding of mathematical concepts in order to provide them with a secure base for further learning. In the planning process, teachers will need to discuss the ways in which students may demonstrate this understanding. The amount of time and experiences dedicated to this stage of learning will vary from student to student.

The learning outcomes in the transferring meaning into symbols stage are more obviously demonstrable and observable. The expectation for students working in this stage is that they have demonstrated understanding of the underlying concepts before being asked to transfer this meaning into symbols. It is acknowledged that, in some strands, symbolic representation will form part of the constructing meaning stage. For example, it is difficult to imagine how a student could construct meaning about the way in which information is expressed as organized and structured data without having the opportunity to collect and represent this data in graphs. In this type of example, perhaps the difference between the two stages is that in the transferring meaning into symbols stage the student will be able to demonstrate increased independence with decreasing amounts of teacher prompting required for them to make connections. Another difference could be that a student's own symbolic representation may be extended to include more conventional methods of symbolic representation.

In the final stage, a number of learning outcomes have been developed to reflect the kind of actions and behaviours that students might demonstrate when applying with understanding. It is important to note that other forms of application might be in evidence in classrooms where there are authentic opportunities for students to make spontaneous connections between the learning that is going on in mathematics and other areas of the curriculum and daily life.

When a continuum for a particular strand is observed as a whole, it is clear how the conceptual understandings and the associated learning outcomes develop in complexity as they are viewed across the phases. In each of the phases, there is also a vertical progression where most learning outcomes identified in the constructing meaning stage of the phase are often described as outcomes relating to the transferring meaning into symbols and applying with understanding stages of the same phase. However, on some occasions, a mathematical concept is introduced in one phase but students are not expected to apply the concept until a later phase. This is a deliberate decision aimed at providing students with adequate time and opportunities for the ongoing development of understanding of particular concepts.

Each of the continuums contains a **notes** section which provides extra information to clarify certain learning outcomes and to support planning, teaching and learning of particular concepts.

Mathematical strands:

At TIS we teach five different mathematical strands. Where possible, mathematics is taught through the unit of inquiry, giving context and an authentic purpose to the acquisition of mathematical concepts, knowledge and skills.

Data handling

Data handling allows us to make a summary of what we know about the world and to make inferences about what we do not know. Data can be collected, organized, represented and summarized in a variety of ways to highlight similarities, differences and trends; the chosen format should illustrate the information without bias or distortion. Probability can be expressed qualitatively by using terms such as "unlikely", "certain" or "impossible". It can be expressed quantitatively on a numerical scale.

Measurement

To measure is to attach a number to a quantity using a chosen unit. Since the attributes being measured are continuous, ways must be found to deal with quantities that fall between numbers. It is important to know how accurate a measurement needs to be or can ever be.

Shape and space

The regions, paths and boundaries of natural space can be described by shape. An understanding of the interrelationships of shape allows us to interpret, understand and appreciate our two-dimensional (2D) and three-dimensional (3D) world.

Pattern and Function

To identify patterns is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called "functions". This builds a foundation for the later study of algebra.

Number

Our number system is a language for describing quantities and the relationships between quantities. For example, the value attributed to a digit depends on its place within a base system. Numbers are used to interpret information, make decisions and solve problems. For example, the operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends on how the result will be used.

Mathematics Strand: Data handling

Data handling allows us to make a summary of what we know about the world and to make inferences about what we do not know. Data can be collected, organized, represented and summarized in a variety of ways to highlight similarities, differences and trends; the chosen format should illustrate the information without bias or distortion. Probability can be expressed qualitatively by using terms such as "unlikely", "certain" or "impossible". It can be expressed quantitatively on a numerical scale.

Overall
expectations Phase 1
Learners will develop an understanding of how the collection and organization of information helps to make sense of the world. They will sort, describe and label objects by attributes and represent information in graphs including pictographs and tally marks. The learners will discuss chance in daily events.

Phase 2
Learners will understand how information can be expressed as organized and structured data and that this can occur in a range of ways. They will collect and represent data in different types of graphs, interpreting the resulting information for the purpose of answering questions. The learners will develop an understanding that some events in daily life are more likely to happen than others and they will identify and describe likelihood using appropriate vocabulary.

Phase 3
Learners will continue to collect, organize, display and analyse data, developing an understanding of how different graphs highlight different aspects of data more efficiently. They will understand that scale can represent different quantities in graphs and that mode can be used to summarize a set of data. The learners will make the connection that probability is based on experimental events and can be expressed numerically.

Phase 4
Learners will collect, organize and display data for the purposes of valid interpretation and communication. They will be able to use the mode, median, mean and range to summarize a set of data. They will create and manipulate an electronic database for their own purposes, including setting up spreadsheets and using simple formulas to create graphs. Learners will understand that probability can be expressed on a scale (0–1 or 0%–100%) and that the probability of an event can be predicted theoretically.

Data Handling - Phase 1

Conceptual Understandings:

1. We collect information to make sense of the world around us.
2. Organizing objects and events helps us to solve problems.
3. Events in daily life involve chance.

Learning Outcomes:	Benchmarks		
]When **constructing meaning** learners: ● understand that sets can be organized by different attributes ● understand that information about themselves and their surroundings can be obtained in different ways ● discuss chance in daily events (impossible, maybe, certain). When **transferring meaning into symbols** learners: ● represent information through pictographs and tally marks ● sort and label real objects by attributes When **applying with understanding** learners: ● create pictographs and tally marks ● create living graphs using real objects and people* ● describe real objects and events by attributes.	**By the end of K1 students are expected to:**	**By the end of K2/3 students are expected to:**	**By the end of Grade 1 students are expected to:**
	- sort and classify real life objects into sets - match one to one - sort, classify and compare objects by attribute	- construct and read simple pictographs - recognize and respond to unpredictability using the terms impossible, maybe and certain - collect and display and interpret data related to their own activities using simple pictographs	- sort, classify and compare by attribute - understands one-to-one correspondence

	Notes
	Units of inquiry will be rich in opportunities for collecting and organizing information. It may be useful for the teacher to provide scaffolds, such as questions for exploration, and the modelling of graphs and diagrams.
	*Living graphs refer to data that is organized by physically moving and arranging students or actual materials in such a way as to show and compare quantities.
	Very young children view the world as a place of possibilities. The teacher should try to introduce practical examples and should use appropriate vocabulary. Discussions about chance in daily events should be relevant to the context of the learners.

Data Handling - Phase 2

Conceptual Understandings:

1. Information can be expressed as organized and structured data.
2. Objects and events can be organized in different ways.
3. Some events in daily life are more likely to happen than others.

Learning Outcomes:	Benchmarks	
When **constructing meaning** learners:	**By the end of Grade 2 students are expected to:**	**By the end of Grade 3 students are expected to:**
● understand that sets can be organized by one or more attributes ● understand that information about themselves and their surroundings can be collected and recorded in different ways ● understand the concept of chance in daily events (impossible, less likely, maybe, most likely, certain). When **transferring meaning into symbols** learners: ● collect and represent data in different types of graphs, for example, tally marks, bar graphs ● represent the relationship between objects in sets using Venn and Carroll diagrams ● express the chance of an event happening using words or phrases (impossible, less likely, maybe, most likely, certain).	- interpret and construct simple pictograms, tally charts, block diagrams and simple tables. - ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity. - ask and answer questions about totalling and comparing categorical data. - use expanded vocabulary such as never and always to describe the likelihood of events.	- interpret and present data using bar charts, pictograms and tables. - solve one-step and two-step questions (for example, 'How many more?' and 'How many fewer?') using information presented in scaled bar charts, pictograms and tables. - place events in order of likelihood - collect data and present it in pictographs and simple bar graphs

When **applying with understanding** learners:	- sorts, classifies and interpret simple pictographs and bar graphs	- interpret data from graphs and graphic organisers using words such as more, fewer, less than, greater than
- collect, display and interpret data for the purpose of answering questions including the use of ICT tools - create a pictograph and sample bar graph of real objects and interpret data by comparing quantities (for example, more, fewer, less than, greater than) - use Venn and Carroll diagrams to explore relationships between data - identify and describe chance in daily events (impossible, less likely, maybe, most likely, certain).	- use words such as more, fewer, less than, greater than - organize data from graphs and diagrams such as Carroll and Venn - predict the outcomes of chance events	

Notes:

An increasing number of computer and web-based applications are available that enable learners to manipulate data in order to create graphs.

Students should have a lot of experience of organizing data in a variety of ways, and of talking about the advantages and disadvantages of each. Interpretations of data should include the information that cannot be concluded as well as that which can. It is important to remember that the chosen format should illustrate the information without bias.

Situations that come up naturally in the classroom, often through literature, present opportunities for discussing probability. Discussions need to take place in which students can share their sense of likelihood in terms that are useful to them.

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<tr><td colspan="3" align="center">Data Handling - Phase 3</td></tr>
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Conceptual Understandings:
1. Data can be collected, organized, displayed and analysed in different ways.
2. Different graph forms highlight different aspects of data more efficiently.
3. Probability can be based on experimental events in daily life.
4. Probability can be expressed in numerical notations.

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<tr><td>Learning Outcomes:</td><td colspan="2" align="center">Benchmarks</td></tr>
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When **constructing meaning** learners:

- understand that data can be collected, displayed and interpreted using simple graphs, for example, bar graphs, line graphs

- understand that scale can represent different quantities in graphs

- understand that the mode can be used to summarize a set of data

- understand that one of the purposes of a database is to answer questions and solve problems

- understand that probability is based on experimental events.

When **transferring meaning into symbols** learners:

- collect, display and interpret data using simple graphs, for example, bar graphs, line graphs

- identify, read and interpret range and scale on graphs

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By the end of Grade 4 students are expected to:

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By the end of Grade 5 students are expected to:

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- interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs.

- solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

- interpret information from pictographs, bar graphs, charts and tables

- construct/create and label graphs and tables to clearly show data

- explore the likelihood of an event happening from likely, unlikely to very unlikely

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- solve comparison, sum and difference problems using information presented in a line graph

- complete, read and interpret information in tables, including timetables.

- compare likelihood of everyday events e.g. rain

- plan and construct probability experiments e.g. colour spinners

- use tree diagrams to express probability

- express probability using simple fractions

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● identify the mode of a set of data ● use tree diagrams to express probability using simple fractions. When **applying with understanding** learners: ● design a survey and systematically collect, organize and display data in pictographs and bar graphs including the use of ICT tools ● select appropriate graph form(s) to display data ● interpret range and scale on graphs ● use probability to determine mathematically fair and unfair games and to explain possible outcomes ● express probability using simple fractions.	- survey groups to find information and to solve problems - use diagrams to express probability	- generate survey questions for finding information and to solve problems - use a column or bar graph to display results of an experiment - construct and label graphs and tables to clearly show data - select appropriate graph forms to display data - interpret information from pictographs, line and bar graphs, charts and tables - understand that scale can represent different quantities in graphs and tables

Notes:

Using data that has been collected and saved is a simple way to begin discussing the mode. A further extension of mode is to formulate theories about why a certain choice is the mode.

Students should have opportunities to connect their work on coordinated and scales to their interpretation of time graphs, ideally with data they've collected.

Students should have the opportunity to use databases, ideally, those created using data collected by the students then entered into a database by the teacher or together.

Situations that come up naturally in the classroom or form part of the units of inquiry present opportunities for students to further develop their understanding of statistics and probability concepts.

Data Handling - Phase 4	

Conceptual Understandings:

1. Data can be presented effectively for valid interpretation and communication.
2. Range, mode, median and mean can be used to analyse statistical data.
3. Probability can be represented on a scale between 0–1 or 0%–100%.
4. The probability of an event can be predicted theoretically.

Learning Outcomes:	Benchmarks

When **constructing meaning** learners:
   ● understand that different types of graphs have special purposes
 	
   ● understand that the mode, median, mean and range can summarize a set of data
 	
   ● understand that probability can be expressed in scale (0–1) or percent (0%–100%)
 	
   ● understand the difference between experimental and theoretical probability
 	
   ● understand that the same data can be represented as a pie chart (circle graph), percentage and fraction
 	
   ● understand the connection between angles and pie charts
 	
When **transferring meaning into symbols** learners:
 	
   ● collect, display and interpret data in pie charts (circle graphs) and line graphs
	By the end of Grade 6 students are expected to:
	
	- interpret and construct pie charts and line graphs and use these to solve problems
	
	- calculate and interpret the mean as an average
	
	- write questionnaires/surveys to collect data
	
	- construct and interpret bar, column, line and pie graphs
	
	- identify the range, mode, median and mean in a set of data
	
	- select appropriate graph forms to display data
	
	- set up a spreadsheet using simple formulas to manipulate data and to create graphs
	
	- explain probability in chance events
	
	- express probability using scale (0–1) or per cent (0%–100%).

	Notes:

● identify, describe and explain the range, mode, median and mean in a set of data ● set up a spreadsheet using simple formulas to manipulate data and to create graphs ● express probability using scale (0–1) or percent (0%–100%) ● display the same piece of data as a pie chart (circle graph), percentage and fraction When **applying with understanding** learners: ● design a survey and systematically collect, record, organize and display the data in a bar graph, pie chart (circle graph), line graph, percentage, fraction including the use of ICT tools ● identify, describe and explain the range, mode, median and mean in a set of data ● create and manipulate an electronic database for their own purposes ● determine the theoretical probability of an event and explain why it might differ from experimental probability	A database is a collection of data, where the data can be displayed in many forms. The data can be changed at any time. A spreadsheet is a type of database where information is set out in a table. Using a common set of data is a good way for students to start to set up their own databases. A unit of inquiry would be an excellent source of common data for student practice. Technology gives us the option of creating a graph at the press of a key. Being able to generate different types of graphs allows learners to explore and appreciate the attributes of each type of graph and its efficacy in displaying the data. Technology also gives us the possibility of rapidly replicating random events. Computer and web-based applications can be used to toss coins, roll dice, and tabulate and graph the results. Learners connect their work on angles, fractions and percentages to the interpretation of pie charts.

Mathematics Strand: Measurement

To measure is to attach a number to a quantity using a chosen unit. Since the attributes being measured are continuous, ways must be found to deal with quantities that fall between numbers. It is important to know how accurate a measurement needs to be or can ever be.

Overall

expectations Phase 1

Learners will develop an understanding of how measurement involves the comparison of objects and the ordering and sequencing of events. They will be able to identify, compare and describe attributes of real objects as well as describe and sequence familiar events in their daily routine.

Phase 2

Learners will understand that standard units allow us to have a common language to measure and describe objects and events, and that while estimation is a strategy that can be applied for approximate measurements, particular tools allow us to measure and describe attributes of objects and events with more accuracy. Learners will develop these understandings in relation to measurement involving length, mass, capacity, money, temperature and time.

Phase 3

Learners will continue to use standard units to measure objects, in particular developing their understanding of measuring perimeter, area and volume. They will select and use appropriate tools and units of measurement, and will be able to describe measures that fall between two numbers on a scale. The learners will be given the opportunity to construct meaning about the concept of an angle as a measure of rotation.

Phase 4

Learners will understand that a range of procedures exists to measure different attributes of objects and events, for example, the use of formulas for finding area, perimeter and volume. They will be able to decide on the level of accuracy required for measuring and using decimal and fraction notation when precise measurements are necessary. To demonstrate their understanding of angles as a measure of rotation, the learners will be able to measure and construct angles.

Measurement - Phase 1

Conceptual Understandings:

1. Measurement involves comparing objects and events.
2. Objects have attributes that can be measured using non-standard units.
3. Events can be ordered and sequenced.

Learning Outcomes:	Benchmarks		
When **constructing meaning** learners: ● understand that attributes of real objects can be compared and described, for example, longer, shorter, heavier, empty, full, hotter, colder ● understand that events in daily routines can be described and sequenced, for example, before, after, bedtime, story-time, today, tomorrow. When **transferring meaning into symbols** learners: ● identify, compare and describe attributes of real objects, for example, longer, shorter, heavier, empty, full, hotter, colder ● compare the length, mass and capacity of objects using non-standard units ● identify, describe and sequence events in their daily routine, for example, before, after, bedtime, story-time, today, tomorrow. When **applying with understanding** learners:	**By the end of K1 students are expected to:**	**By the end of K2/3 students are expected to:**	**By the end of Grade 1 students are expected to:**
	- use everyday language to compare objects by weight, length and size	- understand terminology such as before, after, yesterday, tomorrow - understand simple ordering of events in a day - measure using non-standard units - be familiar with days of the week - order three items by length or height - order two items by weight or capacity	- compare, describe and solve practical problems for length and heights, mass and weight, capacity and volume and time - sequence events in chronological order using language (for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening) - recognise and use language relating to dates, including days of

			the week, months and current Grade
● describe observations about events and objects in real-life situations			- select and use appropriate mathematical equipment
● use non-standard units of measurement to solve problems in real-life situations involving length, mass and capacity.			- recognise the continuity of time and the natural cycles such as day and night
			- use a calendar to identify days and dates

Notes

Learners need many opportunities to experience and quantify measurement in a direct kinesthetic manner. They will develop understanding of measurement by using manipulatives and materials from their immediate environment, for example, containers of different sizes, sand, water, beads, corks and beans.

The pairs of terms: mass and weight, volume and capacity, are used interchangeably at this stage.

Learners move from using and comparing different types of quantities and measures using non-standard units, including discrete (for example, counting) and continuous (for example, liquid) measurement, to using manageable common standard units.

Learners will use the language of measurement, such as long/short, longer/shorter, tall/short, double/half, heavy/light, heavier than, lighter than, full/empty, more than, less than, half, half full, quarter, quicker, slower, earlier, later.

In order to become familiar with standard measures, learners begin to use measuring tools such as a ruler, weighing scales and containers.

	Learners use the language of time to discuss the order of events in their daily lives. Learners may express curiosity about telling time. Teachers are encouraged to respond to these inquiries in an authentic context.

Measurement - Phase 2		

Conceptual Understandings:
1. Standard units allow us to have a common language to identify, compare, order and sequence objects and events.
2. We use tools to measure the attributes of objects and events.
3. Estimation allows us to measure with different levels of accuracy.

Learning Outcomes:	Benchmarks	
When **constructing meaning** learners: ● understand the use of standard and non-standard units to measure, for example, length, mass, capacity, money, time, temperature ● understand that tools can be used to measure ● understand that calendars can be used to determine the date, and to identify and sequence days of the week and months of the Grade ● understand that time is measured using universal units of measure, for example, Grades, months, days, hours, minutes and seconds ● understand that most monetary systems are a base ten system by exploring host and foreign currencies ● understand that time can be recorded using a variety of equivalent methods (analogue clocks, digital clocks, sundials, roman numerals, etc…) When **transferring meaning into symbols** learners:	**By the end of Grade 2 students are expected to:** - find different combinations of coins that equal the same amounts of money - solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change - use non-standard units to measure - understand the use of standard units for measurement using correct vocabulary - estimate and compare length, mass, and volume using standard and non-standard measurement - understands the concept of approximate and actual measurement	**By the end of Grade 3 students are expected to:** - estimate, measure, compare, add and subtract accurately using standard and non-standard measurements (g, kg, m, mm, cm, l, ml) - use appropriate vocabulary when estimating and comparing units of measurement - estimate and measure the perimeter of simple 2-D shapes - tell and write the time to the hour, half hour, quarter hours (to and after) using a 12 hour clock - tell the time using a 24-hour clock - make connections between analogue and digital times

● estimate and measure objects using standard units of measurement: length, mass, capacity, money and temperature and record using >, <, = ● read and write the time to the hour, half hour and quarter hour using analogue and digital clocks ● estimate and compare lengths of time: second, minute, hour, day, week and month.	- match important events to months of the Grade - use non-standard units to measure duration of time - tell and record time to hour and half hour - order, read and write days of the week and months of the Grade	- understand the relationships between equivalent standard units - match temperature to real life experience

When **applying with understanding** learners: ● use standard units of measurement to solve problems in real-life situations involving length, mass, capacity, money and temperature ● use measures of time to assist with problem solving in real-life situations ● compare the duration of events using seconds, minutes, and hours	**Notes:** Using materials from their immediate environment, learners can investigate how units are used for measurement and how measurements vary depending on the unit that is used. Learners will refine their estimation and measurement skills by basing estimates on prior knowledge, measuring the object and comparing actual measurements with their estimations. The comparison of measures includes simple scaling by integers (for example, a given quantity or measure is twice as long or five times as high) and this connects to multiplication.

Measurement - Phase 3		
Conceptual Understandings: 1. Objects and events have attributes that can be measured using appropriate tools. 2. Relationships exist between standard units of measurement.		
Learning Outcomes:	**Benchmarks**	
When constructing meaning learners: ● understand the use of standard units to measure perimeter, area and volume ● understand that measures can fall between numbers on a measurement scale, for example, 3½ kg, between 4 cm and 5 cm ● understand relationships between units, for example, metres, centimetres and millimetres ● understand the relationship between units with a base ten monetary system exploring host and foreign currencies ● understand an angle as a measure of rotation **When transferring meaning into symbols learners:** ● estimate and measure using standard units of measurement: perimeter, area, volume, and money ● describe measures that fall between numbers on a scale	**By the end of Grade 4 students are expected to:**	**By the end of Grade 5 students are expected to:**
	- use formal units to estimate and measure length, area, mass, perimeter, volume and capacity - understand the relationships between equivalent standard units, for example kilometre to metre; hour to minutes; rupees and cents - read an analogue and digital clock to nearest 5 minutes - identify dates and months on a calendar - fill in information on a calendar accurately - understand the duration of time in relation to the calendar - interpret timetables and calendars in relation to familiar events	- estimate and accurately measure length, perimeter, area, volume, capacity, time and mass - recognise and use different units of measure e.g. paces, centimetres, metres - understand the relationships between equivalent standard units (for example, kilometre to metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre) - read and tell time accurately to the minute - analogue and digital - demonstrate an understanding of time sequencing using clocks and calendars

● read and write digital and analogue time on 12-hour and 24-hour clocks.		- understand the concept of elapsed time using seconds, minutes, days and weeks - interpret timetables and calendars in relation to familiar events

When applying with understanding learners:

- use standard units of measurement to solve problems in real-life situations involving perimeter, area, volume, and money

- select appropriate tools and units of measurement

- use timelines in units of inquiry and other real-life situations

- solve problems involving converting hours to minutes; minutes to seconds; Grades to months; weeks to days.

Notes: Grade 4

Learners build on their understanding of place value and decimal notation to record metric measures, including money.

They use multiplication to convert from larger to smaller units.

Perimeter can be expressed algebraically as $2(a + b)$ where a and b are the dimensions in the same unit.

They relate area to arrays and multiplication.

Notes: Grade 5

Learners use their knowledge of place value and multiplication and division to convert between standard units.

Learners calculate the perimeter of rectangles and related composite shapes, including using the relations of perimeter or area to find unknown lengths.
Missing measures questions such as these can be expressed algebraically, for example $4 + 2b = 20$ for a rectangle of sides 2 cm and b cm and perimeter of 20cm.

Learners calculate the area from scale drawings using given measurements.

Learners use all four operations in problems involving time and money, including conversions (for example, days to weeks, expressing the answer as weeks and days).

Notes:

	In order to use measurement more authentically, learners should have the opportunity to measure real objects in real situations. The units of inquiry can often provide these realistic contexts.
	A wide range of measuring tools should be available to the students, for example, rulers, trundle wheels, tape measures, bathroom scales, kitchen scales, timers, analogue clocks, digital clocks, stopwatches and calendars. There are an increasing number of computer and web-based applications available for students to use in authentic contexts.
	Learners can connect counting squares when estimating measures of irregular shapes and the understanding that measures fall between units (fraction).
	Please note that outcomes relating to angles also appear in the shape and space strand.

Measurement - Phase 4

Conceptual Understandings:
1. Accuracy of measurements depends on the situation and the precision of the tool.
2. Conversion of units and measurements allows us to make sense of the world we live in.
3. A range of procedures exists to measure different attributes of objects and events.

Learning Outcomes:	Benchmarks
When **constructing meaning** learners**:**	**By the end of Grade 6 students are expected to:**
- understand procedures for finding area, perimeter and volume - understand the relationships between area and perimeter, between area and volume, and between volume and capacity - understand unit conversions within measurement systems (metric or customary) When **transferring meaning into symbols** learners**:** - develop and describe formulas for finding perimeter, area and volume - use decimal and fraction notation in measurement, for example, 3.2 cm, 1.47 kg, 1½ miles - read and interpret scales on a range of measuring instruments - measure and construct angles in degrees using a protractor - carry out simple unit conversions within a system of measurement (metric or customary).	- use estimation and apply measurement concepts to everyday situations - read and use a 24 hour clock - understand international time zones - solve problems involving converting between units of time - read and interpret timetables and schedules - measure angles in degrees with a protractor - apply concepts of decimals and fractions related to measurement **Notes:** Learners generalize their measuring experiences as they devise procedures and formulas for working out perimeter, area and volume. While the emphasis for understanding is on measurement systems commonly used in the learner's world, it is worthwhile being aware of the existence of other systems and how conversions between systems help us to make sense of them.

When **applying with understanding** learners:	Learners connect conversion (eg. kilometres to metres) to a graphical representation as preparation for understanding linear/proportional graphs.
● select and use appropriate units of measurement and tools to solve problems in real-life situations	They know approximate conversions and are able to tell if an answer is sensible.
● determine and justify the level of accuracy required to solve real-life problems involving measurement	Using the number line, learners use, add and subtract positive and negative integers for measures such as temperature.
● use decimal and fractional notation in measurement, for example, 3.2 cm, 1.47 kg, 1½ miles	Learners could be introduced to compound units for speed, such as kilometer per hour, and apply their knowledge in science or other subjects as appropriate.
● use timetables and schedules (12-hour and 24-hour clocks) in real-life situations	
● determine times worldwide.	

Mathematics Strand: Shape and space

The regions, paths and boundaries of natural space can be described by shape. An understanding of the interrelationships of shape allows us to interpret, understand and appreciate our two-dimensional (2D) and three-dimensional (3D) world.

Overall

expectations Phase 1

Learners will understand that shapes have characteristics that can be described and compared. They will understand and use common language to describe paths, regions and boundaries of their immediate environment.

Phase 2

Learners will continue to work with 2D and 3D shapes, developing the understanding that shapes are classified and named according to their properties. They will understand that examples of symmetry and transformations can be found in their immediate environment.
Learners will interpret, create and use simple directions and specific vocabulary to describe paths, regions, positions and boundaries of their immediate environment.

Phase 3

Learners will sort, describe and model regular and irregular polygons, developing an understanding of their properties. They will be able to describe and model congruency and similarity in 2D shapes. Learners will continue to develop their understanding of symmetry, in particular reflective and rotational symmetry. They will understand how geometric shapes and associated vocabulary are useful for representing and describing objects and events in real-world situations.

Phase 4

Learners will understand the properties of regular and irregular polyhedra. They will understand the properties of 2D shapes and understand that 2D representations of 3D objects can be used to visualize and solve problems in the real world, for example, through the use of drawing and modelling. Learners will develop their understanding of the use of scale (ratio) to enlarge and reduce shapes. They will apply the language and notation of bearing to describe direction and position.

Shape and Space - Phase 1			

Conceptual Understandings:
1. Shapes can be described and organized according to their properties.
2. Objects in our immediate environment have a position in space that can be described according to a point of reference.

Learning Outcomes:	**Benchmarks**		
When constructing meaning learners: ● understand that 2D and 3D shapes have characteristics that can be described and compared using mathematical language ● understand that common language can be used to describe position and direction, for example, inside, outside, above, below, next to, behind, in front of, up, down. **When transferring meaning into symbols learners:** ● sort, describe and compare 2D and 3D shapes ● describe position and direction, for example, inside, outside, above, below, next to, behind, in front of, up, down. **When applying with understanding learners:** ● explore and describe the paths, regions and boundaries of their immediate environment (inside, outside, above, below) and their position (next to, behind, in front of, up, down).	**By the end of K1 students are expected to:**	**By the end of K2/3 students are expected to:**	**By the end of Grade 1 students are expected to:**
	- follow simple directions to move from one place to another or to move objects - identify and describe 2D shapes - sort 2D shapes - show an interest in shape through play	- recognise simple shapes - sort and match simple shapes - explore properties of real life shapes (3D) - identify and describe 2D and 3D shapes - sort and compare 2D and 3D shapes - recognise relationships between 2D and 3D shapes	- recognise, copy and draw points, lines and simple free hand curves - identify basic two-dimensional shapes - identify basic three-dimensional solids and objects - sort geometric objects according to simple descriptions - place and orientate objects according to simple descriptions such as next to, besides, in front of, behind, under, over

	Notes
	Learners need many opportunities to experience shape and space in a direct kinesthetic manner, for example, through play, construction and movement.
	The manipulatives that they interact with should include a range of 3D shapes, in particular the real-life objects with which children are familiar. 2D shapes (plane shapes) are a more abstract concept but can be understood as faces of 3D shapes.
	Learners handle common 2-D and 3-D shapes, naming these and related everyday objects fluently. They recognise these shapes in different orientations and sizes, and know that rectangles, triangles, cuboids and pyramids are not always similar to each other.

Shape and Space - Phase 2		

Conceptual Understandings:
1. Shapes are classified and named according to their properties.
2. Some shapes are made up of parts that repeat in some way.
3. Specific vocabulary can be used to describe an object's position in space.

Learning Outcomes:	Benchmarks	
When **constructing meaning** learners: ● understand that there are relationships among and between 2D and 3D shapes ● understand that 2D and 3D shapes can be created by putting together and/or taking apart other shapes ● understand that examples of symmetry and transformations can be found in their immediate environment ● understand that directions can be used to describe pathways, regions, positions and boundaries of their immediate environment. When **transferring meaning into symbols** learners: ● sort, describe and label 2D and 3D shapes ● create and describe symmetrical and tessellating patterns ● identify lines of reflective symmetry ● represent ideas about the real world using geometric vocabulary and symbols, for example, through oral description, drawing, modelling, labelling ● interpret and create simple directions, describing paths, regions, positions and boundaries of their immediate environment. When **applying with understanding** learners:	**By the end of Grade 2 students are expected to:** - use common terms to describe the properties of familiar two-dimensional and three-dimensional shapes - identify shapes that have symmetry - make models of three-dimensional shapes - identify paths on simple maps by identifying key locations and directions such as left and right - understand that geometric shapes are useful for representing real-world situations	**By the end of Grade 3 students are expected to:** - recognise lines, surfaces and planes, corners and boundaries - analyse and describe the relationships between 2D and 3D shapes - recognise three-dimensional shapes and objects including pyramids, cones and cylinders - arrange a range of geometric shapes such as attribute blocks, into subsets according to simple criteria - recognise and describe symmetry, asymmetry and congruence - apply simple transformations to shapes – flips, turns, slides, enlargements

		- interpret a map and give simple directions using landmarks and symbols
● analyse and use what they know about 3D shapes to describe and work with 2D shapes ● recognize and explain simple symmetrical designs in the environment ● apply knowledge of symmetry to problem-solving situations ● interpret and use simple directions, describing paths, regions, positions and boundaries of their immediate environment.		**Notes** Learners need to understand the properties of 2D and 3D shapes before the mathematical vocabulary associated with shapes makes sense to them. Through creating and manipulating shapes, learners align their natural vocabulary with more formal mathematical vocabulary and begin to appreciate the need for this precision. Learners identify, compare and sort shapes on the basis of their properties and use vocabulary precisely, such as sides, edges, vertices and faces. Learners read and write names for shapes that are appropriate for their word reading and spelling. Learners draw lines and shapes using a straight edge. Learners should work with patterns of shapes, including those in different orientations. Learners use the concept and language of angles to describe 'turn' by applying rotations, including in practical contexts (for example, learners themselves moving in turns, giving instructions to other learners to do so, and programming robots using instructions given in right angles). Learners' knowledge of the properties of shapes is extended at this stage to symmetrical and non-symmetrical polygons and polyhedra. Learners extend their use of the properties of shapes. They should be able to describe the properties of 2-D and 3-D shapes using accurate language, including lengths of lines and acute and obtuse for angles greater or lesser than a right angle. Learners identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle.

	Learners handle and name a wide variety of common 2-D and 3-D shapes including: quadrilaterals and polygons, and cuboids, prisms and cones, and identify the properties of each shape (for example, number of sides, number of faces).

Shape and Space - Phase 3

Conceptual Understandings:
1. Changing the position of a shape does not alter its properties.
2. Shapes can be transformed in different ways.
3. Geometric shapes and vocabulary are useful for representing and describing objects and events in real-world situations.

Learning Outcomes:	Benchmarks	
When **constructing meaning** learners: ● understand the common language used to describe shapes ● understand the properties of regular and irregular polygons ● understand congruent or similar shapes ● understand that lines and axes of reflective and rotational symmetry assist with the construction of shapes ● understand an angle as a measure of rotation ● understand that directions for location can be represented by coordinates on a grid ● understand that visualization of shape and space is a strategy for solving problems. When **transferring meaning into symbols** learners: ● sort, describe and model regular and irregular polygons ● describe and model congruency and similarity in 2D shapes	**By the end of Grade 4 students are expected to:** - recognise patterns in the environment such as tessellations - interpret a map and give directions using compass points and symbols - identify four major compass directions using coordinates - classify three-dimensional shapes according to the number of edges, vertices and faces - use nets to create three-dimensional shapes - use correct language to describe angles, direction, position and transformations in shape. - identify and create symmetrical patterns	**By the end of Grade 5 students are expected to:** - recognise and describe directions of lines as vertical, horizontal and diagonal - identify 3D shape edges, vertices and faces - locate and identify places on maps and diagrams - analyse and describe 2D and 3D shapes, including regular and irregular polygons, using geometric vocabulary - recognise and accurately measure angles - recognize and explain symmetrical and asymmetrical patterns, including tessellation, in the environment

● analyse angles by comparing and describing rotations: whole turn; half turn; quarter turn; north, south, east and west on a compass ● locate features on a grid using coordinates	- identify regular and irregular shapes - identify the properties of 3D and 2D shapes	- describe and draw 3D shapes from various viewpoints
● describe and/or represent mental images of objects, patterns, and paths. When **applying with understanding** learners: ● analyse and describe 2D and 3D shapes, including regular and irregular polygons, using geometric vocabulary ● identify, describe and model congruency and similarity in 2D shapes ● recognize and explain symmetrical and asymmetrical patterns, including tessellation, in the environment ● apply knowledge of transformations to problem-solving situations.	**Notes:** Computer and web-based applications can be used to explore shape and space concepts such as symmetry, angles and coordinates. The units of inquiry can provide authentic contexts for developing understanding of concepts relating to location and directions. Learners continue to classify shapes using geometrical properties, extending to classifying different triangles (for example, isosceles, equilateral, scalene) and quadrilaterals (for example, parallelogram, rhombus, trapezium). Learners compare and order angles in preparation for using a protractor and compare lengths and angles to decide if a polygon is regular or irregular. Learners draw symmetric patterns using a variety of media to become familiar with different orientations of lines of symmetry; and recognise line symmetry in a variety of diagrams, including where the line of symmetry does not dissect the original shape. Learners become accurate in drawing lines with a ruler to the nearest millimetre, and measuring with a protractor. They use conventional markings for parallel lines and right angles. Learners connect decimals and rounding to drawing and measuring straight lines in centimetres, in a variety of contexts (see Measurement strand). Learners use angle sum facts and other properties to make deductions about missing angles and relate these to missing number problems. Learners recognise and use reflection and translation in a variety of diagrams, including continuing to use a 2-D grid and coordinates in the first quadrant. Reflection should be in lines that are parallel to the axes.	

Shape and Space - Phase 4

Conceptual Understandings:
1. Manipulation of shape and space takes place for a particular purpose.
2. Consolidating what we know of geometric concepts allow us to make sense of and interact with our world.
3. Geometric tools and methods can be used to solve problems relating to shape and space.

Learning Outcomes:	Benchmarks
When **constructing meaning** learners:	**By the end of Grade 6 students are expected to:**
- understand the common language used to describe shapes - understand the properties of regular and irregular polyhedra - understand the properties of circles - understand how scale (ratios) is used to enlarge and reduce shapes - understand systems for describing position and direction - understand that 2D representations of 3D objects can be used to visualize and solve problems - understand that geometric ideas and relationships can be used to solve problems in other areas of mathematics and in real life.	- classify two-dimensional shapes using features such as lengths, angles, symmetry, straight or curved sides - visualise three-dimensional objects from two-dimensional views - identify a graphically presented scale on a map or plan as use it as a guide to distance - use compass directions, including NNW to describe relative positions and routes - calculating angles within 2D shapes - use mathematical language to describe properties of shape - understands the relationship between 2D and 3D shape
When **transferring meaning into symbols** learners: analyse, describe, classify and visualize 2D (including circles, triangles and quadrilaterals) and 3D shapes, using geometric vocabulary	**Notes:** Tools such as compasses and protractors are commonly used to solve problems in real-life situations. However, care should be taken to ensure that students have a strong understanding of the concepts embedded in the

- describe lines and angles using geometric vocabulary

- identify and use scale (ratios) to enlarge and reduce shapes

- identify and use the language and notation of bearing to describe direction and position

- draw and translate simple shapes on the coordinate plane, and reflect them in the axes

- create and model how a 2D net converts into a 3D shape and vice versa

- explore the use of geometric ideas and relationships to solve problems in other areas of mathematics

When **applying with understanding** learners:

- use geometric vocabulary when describing shape and space in mathematical situations and beyond

- use scale (ratios) to enlarge and reduce shapes

- apply the language and notation of bearing to describe direction and position

- use 2D representations of 3D objects to visualize and solve problems, for example using drawings or models

problem to ensure meaningful engagement with the tools and full understanding of the solution.

Learners draw shapes and nets accurately, using measuring tools and conventional markings and labels for lines and angles.

Learners describe the properties of shapes and explain how unknown angles and lengths can be derived from known measurements. These relationships might be expressed algebraically for example, $d = 2 \times r$; $a = 180 - (b + c)$.

Learners draw and label a pair of axes in all four quadrants with equal scaling. This extends their knowledge of one quadrant to all four quadrants, including the use of negative numbers.

Learners draw and label parallelograms and rhombuses, specified by coordinates in the four quadrants, predicting missing coordinates using the properties of shapes. These might be expressed algebraically for example, translating vertex (a, b) to $(a - 2, b + 3)$; (a, b) and $(a + d, b + d)$ being opposite vertices of a square of side d.

Illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius

Strand: Pattern and Function

To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called "functions". This builds a foundation for the later study of algebra.

Overall

expectations Phase 1

Learners will understand that patterns and sequences occur in everyday situations. They will be able to identify, describe, extend and create patterns in various ways.

Phase 2

Learners will understand that whole numbers exhibit patterns and relationships that can be observed and described, and that the patterns can be represented using numbers and other symbols. As a result, learners will understand the inverse relationship between addition and subtraction, and the associative and commutative properties of addition. They will be able to use their understanding of pattern to represent and make sense of real-life situations and, where appropriate, to solve problems involving addition and subtraction.

Phase 3

Learners will analyse patterns and identify rules for patterns, developing the understanding that functions describe the relationship or rules that uniquely associate members of one set with members of another set. They will understand the inverse relationship between multiplication and division, and the associative and commutative properties of multiplication. They will be able to use their understanding of pattern and function to represent and make sense of real-life situations and, where appropriate, to solve problems involving the four operations.

Phase 4

Learners will understand that patterns can be represented, analysed and generalized using algebraic expressions, equations or functions. They will use words, tables, graphs and, where possible, symbolic rules to analyse and represent patterns. They will develop an understanding of exponential notation as a way to express repeated products, and of the inverse relationship that exists between exponents and roots. The students will continue to use their understanding of pattern and function to represent and make sense of real-life situations and to solve problems involving the four operations.

Pattern and Function - Phase 1

Conceptual Understandings:
1. Patterns and sequences occur in everyday situations.
2. Patterns repeat and grow.

Learning Outcomes:	Benchmarks		
When **constructing meaning** learners: ● understand that patterns can be found in everyday situations, for example, sounds, actions, objects, nature, digital models. When **transferring meaning into symbols** learners: ● explore and describe patterns in various ways, for example, using words, drawings, symbols, materials, actions, numbers. When **applying with understanding** learners: ● extend and create patterns including the use of ICT tools ● uses ICT tools to make a task easier or repeatable and can identify real-world examples.	**By the end of K1 students are expected to**	**By the end of K2/3 students are expected to:**	**By the end of Grade 1 students are expected to:**
	- identify simple patterns by sorting and classifying - create basic patterns with colour, sound, and actions - articulate simple patterns in their environment	- describe, extend and create patterns and sequences	- create and extend simple and complex patterns - see patterns in numbers to 100 - identify odd and even numbers - use language for sorting and classifying - describe the position of objects (in front of, behind, under, over)
	Notes The world is filled with patterns and there will be many opportunities for learners to make this connection across the curriculum. A range of manipulatives can be used to explore patterns including pattern blocks, attribute blocks, colour tiles, calculators, number charts, beans and buttons.		

Pattern and Function - Phase 2		

Conceptual Understandings:
1. Whole numbers exhibit patterns and relationships that can be observed and described.
2. Patterns can be represented using numbers and other symbols.

Learning Outcomes:	Benchmarks	
When **constructing meaning** learners: ● understand that patterns can be found in numbers, for example, odd and even numbers, skip counting ● understand the inverse relationship between addition and subtraction ● understand the associative and commutative properties of addition. When **transferring meaning into symbols** learners: ● represent patterns in a variety of ways, for example, using words, drawings, symbols, materials, actions, numbers ● describe number patterns, for example, odd and even numbers, skip counting. When **applying with understanding** learners: ● extend and create patterns in numbers, for example, odd and even numbers, skip counting ● use number patterns to represent and understand real-life situations ● use the properties and relationships of addition and subtraction to solve problems.	**By the end of Grade 2 students are expected to:**	**By the end of Grade 3 students are expected to:**
	- observe and describe whole number patterns to 100 - observe and describe patterns in real life - understand the inverse relationship between addition and subtraction - apply an appropriate strategy to solve problems - transfer their knowledge to real life situations	- use numbers and symbols to represent patterns - use maths language to describe or explain properties of addition and subtraction - describe how basic facts knowledge applies to larger numbers - begin to use the inverse relationship for simple addition, subtraction, multiplication and division - demonstrate understanding of patterns in real life situations using this to solve problems involving addition, subtraction, multiplication and division

	Notes
	Students will apply their understanding of pattern to the numbers they already know. The patterns they find will help to deepen their understanding of a range of number concepts.

Four-function calculators can be used to explore number patterns.

Learners will recognize the commutative and associative properties of addition are patterns in the number system. See number strand. |

Pattern and Function - Phase 3

Conceptual Understandings:
1. Functions are relationships or rules that uniquely associate members of one set with members of another set.
2. By analysing patterns and identifying rules for patterns it is possible to make predictions.

Learning Outcomes:	Benchmarks	
When **constructing meaning** learners: ● understand that patterns can be analysed and rules identified ● understand that multiplication is repeated addition and that division is repeated subtraction ● understand the inverse relationship between multiplication and division ● understand the associative and commutative properties of multiplication. When **transferring meaning into symbols** learners: ● describe the rule for a pattern in a variety of ways ● represent rules for patterns using words, symbols and tables ● identify a sequence of operations relating one set of numbers to another set. When **applying with understanding** learners: ● select appropriate methods for representing patterns, for example using words, symbols and tables	**By the end of Grade 4 students are expected to:**	**By the end of Grade 5 students are expected to:**
	- understand that number patterns are sequenced and that multiples form patterns - recognise prime and composite numbers - use maths language to describe strategies (for example the pattern of odds/evens, skip counting and multiples). - understand the inverse relationship for addition, subtraction, multiplication and division - competently double and halve numbers	- describe the relationship of number patterns (prime numbers, factors, odds/evens, skip counting and multiples) to solve problems - analyse and explain the use of patterns in problem solving - represent and make sense of real life situations to solve problems involving the four operations - develop an awareness of negative numbers
	Notes: Patterns are central to the understanding of all concepts in mathematics. They are the basis of how our number system is organized. Searching for, and identifying, patterns helps us to see relationships, make generalizations,	

use number patterns to make predictions and solve problems	and is a powerful strategy for problem solving. Functions develop from the study of patterns and make it possible to predict in mathematics problems.
use the properties and relationships of the four operations to solve problems.	
use ICT tools to demonstrate basic concepts related to automation, patterns and algorithmic	

Pattern and Function - Phase 4

Conceptual Understandings:
1. Patterns can often be generalized using algebraic expressions, equations or functions.
2. Exponential notation is a powerful way to express repeated products of the same number.

Learning Outcomes:	Benchmarks
When constructing meaning learners:	**By the end of Grade 6 students are expected to:**
● understand that patterns can be generalized by a rule ● understand exponents as repeated multiplication ● understand the inverse relationship between exponents and roots ● understand that patterns can be represented, analysed and generalized using tables, graphs, words, and, when possible, symbolic rules. **When transferring meaning into symbols learners:**	- recognise the repetitive rule of patterns and use it to solve real-life problems - understand repeated multiplication - be able to use and understand index notation e.g. $1,000,000 = 10\times10\times10\times10\times10\times10$ - use tables and graphs to represent patterns - skip count forward and backward from various starting points using multiples of 2, 3, 4, 5, 10 and 100
● represent the rule of a pattern by using a function ● analyse pattern and function using words, tables and graphs, and, when possible, symbolic rules. **When applying with understanding learners:** ● select appropriate methods (including ICT tools) to analyse patterns and identify rules ● use functions to solve problems	**Notes:** Algebra is a mathematical language using numbers and symbols to express relationships. When the same relationship works with any number, algebra uses letters to represent the generalization. Letters can be used to represent the quantity.

● explore or solve problems by selecting technology for modeling and displaying algorithmic thinking	

Mathematics Strand: Number

Our number system is a language for describing quantities and the relationships between quantities. For example, the value attributed to a digit depends on its place within a base system. Numbers are used to interpret information, make decisions and solve problems. For example, the operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends on how the result will be used.

Overall

expectations Phase 1

Learners will understand that numbers are used for many different purposes in the real world. They will develop an understanding of one-to-one correspondence and conservation of number, and be able to count and use number words and numerals to represent quantities.

Phase 2

Learners will develop their understanding of the base 10 place value system and will model, read, write, estimate, compare and order numbers to hundreds or beyond. They will have automatic recall of addition and subtraction facts and be able to model addition and subtraction of whole numbers using the appropriate mathematical language to describe their mental and written strategies. Learners will have an understanding of fractions as representations of whole-part relationships and will be able to model fractions and use fraction names in real-life situations.

Phase 3

Learners will develop the understanding that fractions and decimals are ways of representing whole-part relationships and will demonstrate this understanding by modelling equivalent fractions and decimal fractions to hundredths or beyond. They will be able to model, read, write, compare and order fractions, and use them in real-life situations. Learners will have automatic recall of addition,

subtraction, multiplication and division facts. They will select, use and describe a range of strategies to solve problems involving addition, subtraction, multiplication and division, using estimation strategies to check the reasonableness of their answers.

Phase 4

Learners will understand that the base 10 place value system extends infinitely in two directions and will be able to model, compare, read, write and order numbers to millions or beyond, as well as model integers. They will develop an understanding of ratios. They will understand that fractions, decimals and percentages are ways of representing whole-part relationships and will work towards modelling, comparing, reading, writing, ordering and converting fractions, decimals and percentages. They will use mental and written strategies to solve problems involving whole numbers, fractions and decimals in real-life situations, using a range of strategies to evaluate reasonableness of answers.

Number - Phase 1

Conceptual Understandings:
1. Numbers are a naming system.
2. Numbers can be used in many ways for different purposes in the real world.
3. Numbers are connected to each other through a variety of relationships.
4. Making connections between our experiences with numbers can help us to develop number sense.

Learning Outcomes:	Benchmarks		
When **constructing meaning** learners: ● understand one-to-one correspondence	**By the end of K1 students are expected to:**	**By the end of K2/3 students are expected to:**	**By the end of Grade 1 students are expected to:**
● understand that, for a set of objects, the number name of the last object counted describes the quantity of the whole set ● understand that numbers can be constructed in multiple ways, for example, by combining and partitioning ● understand conservation of number*	- recognise numbers as opposed to letters in shared reading - recognise numbers through oral games	- count objects on a one to one basis to 20 - play games expressing numbers	- recognize, count, order and write numbers beyond 20 - count in 2s and 10s

● understand the relative magnitude of whole numbers ● recognize groups of zero to five objects without counting (subitizing**) ● understand whole-part-part relationships ● use the language of mathematics to compare quantities, for example, more, less, first, second. When **transferring meaning into symbols** learners: ● connect number names and numerals to the quantities they represent. When **applying with understanding** learners: ● count to determine the number of objects in a set ● use number words and numerals to represent quantities in real-life situations ● use the language of mathematics to compare quantities in real-life situations, for example, more, less, first, second ● subitize in real-life situations ● use simple fraction names in real-life situations ● use counting to solve simple real-life problems	- use one-to-one correspondence up to 10 - orally count to 10	- read, write and order numbers to 10 - recognise first and last through games and visuals - show understanding that numbers can be counted forward and backwards	- use ordinal numbers to describe position from first to twentieth - identify when two sets are of equal or different size - use mathematical language - use/apply math in practical tasks and in real life problems - use concrete materials or number lines to model addition by aggregation (grouping together) to 20 - use concrete materials or number lines to model subtraction by disaggregation (moving apart) to 20 add and subtract by counting forward and backward using a number line.

	Notes
	*To conserve, in mathematical terms, means the amount stays the same regardless of the arrangement.
	** The ability to instantaneously recognise the number of objects in a small group without the need to count them.
	Learners who have been encouraged to select their own apparatus and methods, and who become accustomed to discussing and questioning their work, will have confidence in looking for alternative approaches when an initial attempt is unsuccessful.
	Estimation is a skill that will develop with experience and will help children gain a "feel" for numbers. Children must be given the opportunity to check their estimates so that they are able to further refine and improve their estimation skills.
	There are many opportunities in the units of inquiry and during the school day for students to practise and apply number concepts authentically.
	Learners practise counting (1, 2, 3…), ordering (for example, first, second, third…), and to indicate a quantity (for example, 3 apples, 2 centimetres), including solving simple concrete problems, until they are fluent. They practise counting as reciting numbers and counting as enumerating objects, and counting in twos and tens from different multiples to develop their recognition of patterns in the number system.
	Learners combine and increase numbers, counting forwards and backwards. Learners should become comfortable with the ideas of 1 more, 2 more, 1 less, 2 less, and doubles (2+2, 3+3).
	They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, and part-part-whole so that learners develop the concept of addition and

50

	subtraction and are enabled to use these operations flexibly. They should understand number bonds to 10 as whole-part-part relationships. Through grouping and sharing small quantities, learners begin to understand the foundation for multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities. They make connections between number patterns and counting in twos and tens.

Number - Phase 2		

Conceptual Understandings:
1. The base 10 place value system is used to represent numbers and number relationships.
2. Fractions are ways of representing whole-part relationships.
3. The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems.
4. Number operations can be modelled in a variety of ways.
5. There are many mental methods that can be applied for exact and approximate computations.

Learning Outcomes:	Benchmarks	
When **constructing meaning** learners:	**By the end of Grade 2 students are expected to:**	**By the end of Grade 3 students are expected to:**
- model numbers to hundreds or beyond using the base 10 place value system** - estimate quantities to 100 or beyond - model simple fraction relationships - use the language of addition and subtraction, for example, add, take away, plus, minus, sum, difference - model addition and subtraction of whole numbers - develop strategies for memorizing addition and subtraction number facts - estimate sums and differences - understand situations that involve multiplication and division, using materials, arrays, repeated addition, and mental methods	- count by ones forwards and backwards from various starting points between 0 and 100 - identify a fraction as part of a whole with equal size parts, half and quarter - make models of, or sketch simple fractions of a whole - read, record, interpret and order numbers to 100 - know numbers before and after a given two-digit number up to 100	- count, name and order numbers over 100 - read, write and model numbers up to 999 - understand place value of numbers to hundreds - skip count by 2s, 5s and 10s from 0 to 100 - recognise and form patterns based on simple criteria e.g. odd and even numbers - understand money in base ten system

● model addition and subtraction of fractions with the same denominator.	- choose appropriately from strategies for subtraction and addition including counting up and down	- carry out simple money calculations using local currency to 100
When **transferring meaning into symbols** learners:	- count from 0 by 2s, 5s and 10s to a given target	- describe simple fractions ½, 1/3 and ¼ in terms of equal parts of a whole
● read and write whole numbers up to hundreds or beyond	- identify the number 1 and 10 before and 1 and 10 after	- add and subtract up to two digit numbers by regrouping
● read, write, compare and order cardinal and ordinal numbers	- understand place value to 10s	- mentally compute simple addition and subtraction calculations using bridging, estimation and doubling
● describe mental and written strategies for adding and subtracting two-digit numbers		- explore the patterns of factors from 0 to 10
When **applying with understanding** learners:		- memorize basic facts 2, 5, 10 for multiplication application
● use whole numbers up to hundreds or beyond in real-life situations		- estimate answers to equations to 100
● use cardinal and ordinal numbers in real-life situations		- use mathematical language
● use fast recall of addition and subtraction number facts in real-life situations		- read, write and order both cardinal and ordinal numbers
● use fractions in real-life situations		- describe and calculate simple division as sharing or regrouping.
● use a variety of mental and written strategies for addition and subtraction of two-digit numbers or beyond in real-life situations		- use strategies such as doubling, building to 10 and intuitive strategies to solve addition and subtract
● select an appropriate method for solving a problem, for example, mental estimation, mental or written strategies, or by using a calculator		
● use strategies to evaluate the reasonableness of answers.		
● identify a problem, break it apart, and select appropriate tools to explore and find solutions including the use ICT.		

Notes

**Modelling involves using concrete materials to represent numbers or number operations, for example, the use of pattern blocks or fraction pieces to

represent fractions and the use of base 10 blocks to represent number operations.

Students need to use numbers in many situations in order to apply their understanding to new situations. In addition to the units of inquiry, children's literature also provides rich opportunities for developing number concepts.

To be useful, addition and subtraction facts need to be recalled automatically. Research clearly indicates that there are more effective ways to do this than "drill and practice". Above all, it helps to have strategies for working them out. Counting on, using doubles and using 10s are good strategies, although learners frequently invent methods that work equally well for themselves.

Learners develop efficient mental methods, for example, using commutativity and associativity and multiplication and division facts (for example, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts (for example, $30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).

Difficulties with fractions can arise when fractional notation is introduced before students have fully constructed meaning about fraction concepts.

Learners should partition numbers in different ways (for example, $23 = 20 + 3$ and $23 = 10 + 13$) to support subtraction. They become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that emphasise the value of each digit in two-digit numbers. They begin to understand zero as a place holder.

Learners work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, $40 \div 2 = 20$, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning.

Learners use their understanding of place value and partitioning, and practise using a variety of strategies, such as columnar addition and subtraction with increasingly large numbers up to three digits.

Learners solve simple contextual problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits; 12

	sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Number - Phase 3

Conceptual Understandings:
1. The base 10 place value system can be extended to represent magnitude.
2. Fractions and decimals are ways of representing whole-part relationships.
3. The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems.
4. Even complex operations can be modelled in a variety of ways, for example, an algorithm is a way to represent an operation.

Learning Outcomes:	Benchmarks	
When **constructing meaning** learners:	**By the end of Grade 4 students are expected to:**	**By the end of Grade 5 students are expected to:**
- model numbers to thousands or beyond using the base 10 place value system - model equivalent fractions - use the language of fractions, for example, numerator, denominator - model decimal fractions to hundredths or beyond - model multiplication and division of whole numbers - use the language of multiplication and division, for example, factor, multiple, product, quotient, prime numbers, composite number - model addition and subtraction of fractions with related denominators*** - model addition and subtraction of decimals. When **transferring meaning into symbols** learners:	- use place value to read, write, record and interpret numbers beyond 1000 - use place value to order numbers up to 9999. - use place value to explain the value of tenths in relation to whole numbers - round to the nearest tens/ hundreds/thousands - use mental and intuitive strategies to solve addition and subtraction problems	- use place value to determine the size and order of whole numbers to tens of thousands - use place value to determine the size and order of decimals to hundredths - round numbers up and down to the nearest unit, ten, hundred or thousand - develop fraction notation and compare simple common fractions e.g. 2/3 < 3/4 using physical models - estimate the results of computations

- read, write, compare and order whole numbers up to thousands or beyond

- develop strategies for memorizing addition, subtraction, multiplication and division number facts

- read, write, compare and order fractions

- read and write equivalent fractions

- read, write, compare and order fractions to hundredths or beyond

- describe mental and written strategies for multiplication and division

When **applying with understanding** learners:

- use whole numbers up to thousands or beyond in real-life situations

- use fast recall of multiplication and division number facts in real-life situations

- use decimal fractions in real-life situations

- use mental and written strategies for multiplication and division in real-life situations

- select an efficient method for solving a problem, for example, mental estimation, mental or written strategies, or by using a calculator

- use strategies to evaluate the reasonableness of answers

- add and subtract fractions with related denominators in real-life situations

- add and subtract decimals in real-life situations, including money

- estimate sum, difference, product and quotient in real-life situations, including fractions and decimals

- solve multiplication problems using strategies such as skip counting and building up from known facts.

- demonstrate multiplication and division of whole numbers.

- make models of and name fractions of a whole including representation of more than one part

- use of mathematical symbols <, > =,

- use the language of fractions – numerator, denominator

- understand that addition and subtraction are inverse operations.

- solve addition and subtraction problems involving regrouping, up to 4 digits.

- solve multiplication operations involving 2 and 3 digit numbers by 1 digit numbers

- compute with numbers using all 4 operations

- automatically recall multiplication facts to 12x12

- devise and use written methods for whole number addition and subtraction; multiplication and division by single digits and multiples

- use algorithms for addition and subtraction of numbers to two decimal places including money

- add and subtract simple common fractions with the assistance of physical models

Notes

Modelling using manipulatives provides a valuable scaffold for constructing meaning about mathematical concepts. There should be regular opportunities for learners to work with a range of manipulatives and to discuss and negotiate their developing understandings with others.

***Examples of related denominators include halves, quarters (fourths) and eighths. These can be modelled easily by folding strips or squares of paper.

● identify a problem, break it apart, and select appropriate tools to explore a variety of solutions including the use ICT.	The interpretation and meaning of remainders can cause difficulty for some learners. This is especially true if calculators are being used. For example, $67 \div 4 = 16.75$. This can also be shown as 16¾ or 16 r3. Learners need practice in producing appropriate answers when using remainders. For example, for a school trip with 25 students, only buses that carry 20 students are available. A remainder could not be left behind, so another bus would be required!
	Calculator skills must not be ignored. All answers should be checked for their reasonableness.
	By reflecting on and recording their findings in mathematics learning logs, students begin to notice patterns in the numbers that will further develop their understanding.
	They should go beyond the [0, 1] interval, including relating this to measure. (see Measurement and Shape and Space strands)
	They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far.
	Learners make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities. Learners use factors and multiples to recognise equivalent fractions and simplify where appropriate (for example, $6/9 = 2/3$ or $1/4 = 2/8$).
	Learners understand throughout that decimals and fractions are different ways of expressing numbers and proportions.
	They practise counting using simple fractions and decimals, both forwards and backwards.
	Learners recognise and describe linear number sequences (for example, 3, 31/2, 4, 41/2...), including those involving fractions and decimals, and find the term-to-term rule in words (for example, add 1/2).

	Learners interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = 98/4 = 24$ r $2 = 24\frac{1}{2} = 24.5 \approx 25$).
	Learners use multiplication and division as inverses to support the introduction of ratio in Grade 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.
	Learners understand throughout that percentages, decimals and fractions are different ways of expressing proportions.
	Learners go beyond the measurement and money models of decimals, for example, by solving puzzles involving decimals.

Number - Phase 4	
Conceptual Understandings: 1. The base 10 place value system extends infinitely in two directions. 2. Fractions, decimal fractions and percentages are ways of representing whole-part relationships. 3. For fractional and decimal computation, the ideas developed for whole-number computation can apply. 4. Ratios are a comparison of two numbers or quantities.	

Learning Outcomes:	Benchmarks
	By the end of Grade 6 students are expected to:
When **constructing meaning** learners: ● model numbers to millions or beyond using the base 10 place value system ● model ratios ● model integers in appropriate contexts ● model exponents and square roots ● model improper fractions and mixed numbers ● simplify fractions using manipulatives ● model decimal fractions to thousandths or beyond ● model percentages ● understand the relationship between fractions, decimals and percentages	- demonstrate understanding of whole numbers up to millions using place value and models e.g. number lines - add and subtract with and without regrouping including long multiplication and division - be aware of methods to divide by double digit numbers and divide by single digits into a 7 digit figure. - multiply by single and double digit s to produce a 7 digit product - understand that subtraction is the inverse of addition and multiplication of division - demonstrate understanding of decimals to hundredths using models e.g. number lines - create and describe patterns using calculators including multiples and squares

- model addition, subtraction, multiplication and division of fractions

- model addition, subtraction, multiplication and division of decimals.

When **transferring meaning into symbols** learners:

- read, write, compare and order whole numbers up to millions or beyond

- read and write ratios

- read and write integers in appropriate contexts

- read and write exponents and square roots

- convert improper fractions to mixed numbers and vice versa

- simplify fractions in mental and written form

- read, write, compare and order decimal fractions to thousandths or beyond

- read, write, compare and order percentages

- convert between fractions, decimals and percentages.

When **applying with understanding** learners:
- use whole numbers up to millions or beyond in real-life situations

- use ratios in real-life situations

- use integers in real-life situations

- convert improper fractions to mixed numbers and vice versa in real-life situations

- model and identify integers, positive and negative on a number line

- use brackets to show order of operation in simple cases

- find equivalent fractions, multiples and fractions of fractions and perform more complex addition and subtraction using fraction models including a number line

- understand the relationship between fractions, decimals and percentages

- perform addition and subtraction with fractions with unlike denominators

- model improper fractions and mixed numbers

- estimate and make approximations in real life situations involving decimals, fractions and percentages

- use mathematical language

- model ratios

- model exponents and square roots.

Notes:

It is not practical to continue to develop and use base 10 materials beyond 1,000. Learners should have little difficulty in extending the place value system once they have understood the grouping pattern up to 1,000. There are a number of websites where virtual manipulatives can be utilized for working with larger numbers.

Estimation plays a key role in checking the feasibility of answers. The method of multiplying numbers and ignoring the decimal point, then adjusting the answer by counting decimal places, does not give the learner an

- simplify fractions in computation answers

- use fractions, decimals and percentages interchangeably in real-life situations

- select and use an appropriate sequence of operations to solve word problems

- select an efficient method for solving a problem: mental estimation, mental computation, written algorithms, by using a calculator

- use strategies to evaluate the reasonableness of answers

- use mental and written strategies for adding, subtracting, multiplying and dividing fractions and decimals in real-life situations.

- estimate and make approximations in real-life situations involving fractions, decimals and percentages.

understanding of why it is done. Application of place value knowledge must precede this application of pattern.

Measurement is an excellent way of exploring the use of fractions and decimals and their interchange.

Students should be given many opportunities to discover the link between fractions and division.

A thorough understanding of multiplication, factors and large numbers is required before working with exponents.

Learners round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Learners practise, use and understand the addition and subtraction of fractions with different denominators by identifying equivalent fractions with the same denominator. They should start with fractions where the denominator of one fraction is a multiple of the other (for example, ½ +1/8 = 5/8) and progress to varied and increasingly complex problems.

Learners use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity (for example, if 1/4 of a length is 36 cm, then the whole length is $36 \times 4 = 144$ cm).

Learners also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations.